Aircraft Control Design Using Improved Time-Domain Stability Robustness Bounds

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This paper addresses the issue of "conservatism" in the time-domain stability robustness bounds obtained by the Lyapunov approach. A state transformation is employed to improve the upper bounds on the linear time-varying perturbation of an asymptotically stable linear time-invariant system for robust stability. This improvement is due to the variance of the conservatism of the Lyapunov stability condition with respect to the basis of the vector space in which the Lyapunov function is constructed. The proposed analysis is applied to a VTOL aircraft control example to provide a constant linear state feedback control that is stability robust for the full range of parameter variations given.

Nomenclature

R^α = real vector space of dimension α
$\rightarrow$ = belongs to
$\lambda[\cdot]$ = eigenvalues of the matrix $[\cdot]$
$\sigma[\cdot]$ = singular value of the matrix $[\cdot]\{\lambda([\cdot][\cdot]T)\}^{\frac{1}{2}}$
$[\cdot]_s$ = symmetric part of a matrix $[\cdot]$
$|(\cdot)|$ = modulus of the entry $(\cdot)$
$[\cdot]_m$ = modulus matrix = matrix with modulus entries
$\forall$ = for all

I. Introduction

THE analysis of stability robustness of a linear time-invariant system subject to linear perturbations (parameter variations) has attracted much attention for quite some time.[1-5] In this analysis, one can consider two types of linear perturbations, namely, time varying and time invariant, that clearly influence the analysis. Even though in many applications the parameter variations can be considered time invariant (or may be very slowly varying), there are also applications, such as the VTOL aircraft control example considered in Narendra and Tripathi[6] in which the parameters are time varying. Much of the published literature in the *frequency domain* stability robustness analysis, including the use of transformation, treats the time-invariant case.[7,8] In the same vein, there is a considerable amount of literature available in the *time domain* on obtaining stability regions and tolerable perturbations for time-invariant perturbations using eigenvalue (Hurwitz invariance) analysis.[9,10] However, the time-varying case, in the time domain, is known to be best handled by Lyapunov stability analysis.

This paper deals primarily with the aspect of stability robustness analysis for time-varying perturbations using the Lyapunov approach. In this area, iterative algorithms are presented to obtain stability robustness conditions,[11,12] but these conditions are implicit in nature. Explicit bounds on the perturbation, which are easy to use, have been presented by Chang and Peng,[13] Patel et al.,[14] and Patel and Toda.[15] In Ref. 13, bounds on the norm of the perturbation matrix are presented, while Ref. 15 extends this work to present element bounds. Recently, by taking advantage of the structural information on the nominal as well as the perturbation matrices, improved measures of stability robustness have been presented by Yedavalli.[16,17] In this paper, a method to further reduce the conservatism of the element bounds (for structural perturbation) is proposed by using a state transformation. This reduction in conservatism is obtained by exploiting the variance of the "Lyapunov criterion conservatism" with respect to the basis of the vector space in which the function is constructed. Of course, the use of a transformation in determining the Lyapunov function that best serves the purpose at hand is not new. In fact, transformation was employed by Patel and Toda,[15] but the bounds obtained were not better than the ones before transformation. Similarly, Siljak[18,19] employed a transformation on Lyapunov equations to get better estimates of interconnection parameters in the context of interconnected (decentralized) system stability studies. However, the results of this paper are meant for the parameter variation problem of a centralized system and they have different implications in comparison with Siljak's results.

The paper is organized as follows. In Sec. II the results of Ref. 17 are briefly reviewed and the details of the state transformation are presented. Section III illustrates the use of the proposed approach by considering the VTOL aircraft control example[6] and, finally, Sec. IV offers some concluding remarks on the scope of the proposed method.

II. Improvement of Robustness Bounds by State Transformation

In this section, we briefly review the upper bounds for robust stability presented in Ref. 17 for "structured" (elemental) perturbations. Structured perturbations are those for which the magnitude bounds on the individual matrix elements are known for a given model structure. Then, the state transformation details are presented and its use in improving the bound is illustrated.

Consider the following linear dynamic system

$$\dot{x}(t) = A(t)x(t) \tag{1a}$$

$$= [A_0 + E(t)]x(t) \tag{1b}$$

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where $x(t) \to R^n$ is the state vector. A_0 is the $n \times n$ nominally asymptotically stable matrix and $E(t)$ is the "error" matrix. In the case of a structured perturbation, the elements of $E(t)$ are such that

$$\max_{t \to [t_0, \infty)} |E_{ij}(t)| = \epsilon_{ij} \quad \text{and} \quad \epsilon = \max_{\forall i,j} \epsilon_{ij} \tag{2}$$

In Ref. 17, it is shown that the system of Eq. (1) [with Eq. (2)] is asymptotically stable if

$$\epsilon_{ij} < \frac{1}{\sigma_{\max}[P_m U_e]_s} \cdot U_{eij} = \mu U_{eij} \tag{3a}$$

for all $U_{eij} \neq 0$, $i, j = 1, \ldots, n$, where P satisfies the Lyapunov matrix equation

$$PA_0 + A_0^T P + 2I_n = 0 \tag{3b}$$

and

$$U_{eij} \triangleq \epsilon_{ij}/\epsilon \quad (\text{thus } 0 \le U_{eij} \le 1) \tag{3c}$$

Simple examples illustrating this bound and the role of matrix U_e in utilizing the structural information about the error matrix are given in Ref. 17.

State Transformation and Its Implications on Bounds

It may be easily shown that the linear system of Eq. (1) is stable (or asymptotically stable) if and only if the system

$$\dot{\hat{x}}(t) = \hat{A}(t)\hat{x}(t) \tag{4a}$$

where

$$\hat{x}(t) = Q^{-1}x(t), \hat{A}(t) = Q^{-1}A(t)Q \tag{4b}$$

and Q a nonsingular time-invariant $n \times n$ matrix, is stable (or asymptotically stable).

Even though the proof of this result is quite straightforward, it is to be emphasized that it is not based on the standard eigenvalue argument, as we are dealing with a time-varying case.

The implication of this theorem is, of course, important in the proposed analysis here. It means that, to investigate the stability of a linear system of Eq. (1), one can transform it, by a linear map, to a different coordinate frame and derive the stability robustness condition in the new (transformed) coordinates. However, realizing that in doing so even the perturbation gets transformed, we do make an inverse transformation to eventually give a bound on the original perturbation and show with the help of examples that it is indeed possible to give improved bounds on the original perturbation.

In sequel, we consider a diagonal transformation matrix Q. (The use of a general nondiagonal transformation matrix is under study.) Let

$$Q = \text{diag}[q_1, q_2, \ldots, q_n] \quad q_i \neq 0, \quad i = 1, 2, \ldots, n \tag{5}$$

Then

$$\hat{A}(t) = \hat{A}_0 + \hat{E}(t) = \begin{bmatrix} a_{11} + e_{11}(t) q_2/q_1 & (a_{12} + e_{12}(t)) & \cdots & q_n/q_1(a_{1n} + e_{1n}(t)) \\ q_1/q_2(a_{21} + e_{21}(t)) & a_{22} + e_{22}(t) & \cdots & q_n/q_2(a_{2n} + e_{2n}(t)) \\ \vdots & \cdots & \cdots & \vdots \\ q_1/q_n(a_{n1} + e_{n1}(t)) & q_2/q_n(a_{n2} + e_{n2}(t)) & \cdots & a_{nn} + e_{nn}(t) \end{bmatrix} \tag{6a}$$

where

$$\hat{A}_0 = Q^{-1}A_0 Q \quad \text{and} \quad \hat{E}(t) = Q^{-1}E(t)Q \tag{6b}$$

Correspondingly, we get [17]

$$\hat{\epsilon}_{ij} = \left|\frac{q_j}{q_i}\right| \epsilon_{ij} \text{ and } \hat{\epsilon} = \max_{i,j} \hat{\epsilon}_{ij} \tag{6c}$$

and

$$\hat{U}_{eij} = \hat{\epsilon}_{ij}/\hat{\epsilon} \tag{6d}$$

It may be seen that Eq. (6a) is similar in form to the "weighted norm" matrix, which has been used successfully in the frequency domain to reduce the conservatism of the stability robustness condition.[20-23]

In general, stability robustness conditions of the type given in Eq. (3) may be used in two different cases as follows:

1) *Given the perturbation ranges* ϵ_{ij}:

In this case, since the left-hand side (LHS) of Eq. (3) is known, the condition of Eq. (3) is used to check the stability of the perturbed system [Eq. (1)], as well as the corresponding conservatism of the condition of Eq. (3). The VTOL aircraft control example considered in Sec. III falls into this category.

2) *The perturbation ranges* ϵ_{ij} *are not known*:

In this case, Eq. (3) is simply treated as specifying a bound on the elemental perturbation ranges ϵ_{ij}. Note that even when ϵ_{ij} on the LHS of Eq. (3) are not explicitly known, it is still possible to specify the U_{eij} elements because U_{eij} [by the definition of Eq. (3c)] has the meaning of the relative magnitudes of ϵ_{ij} compared to the maximum perturbation one expects in A_0. Recall (from Ref. 17) that in the absence of any explicit and relative information on ϵ_{ij}, one can take $U_{eij} = 1$, thereby accounting for the worst-case situation.

This type of delineation is useful in arriving at a transformation appropriate for the purpose of either case 1 or 2. We illustrate these situations by means of simple examples.

Case 1: Left-Hand Side of Eq. (3) Is Known (Checking Stability and Conservatism)

For this case, in the transformed coordinates, the stability condition of Eq. (3) becomes

$$\hat{\epsilon}_{ij} < \hat{\mu}\hat{U}_{eij} \tag{7a}$$

or

$$\hat{\epsilon} < \hat{\mu} \tag{7b}$$

where

$$\hat{\mu} = 1/\sigma_{\max}[\hat{P}_m \hat{U}_e]_s \tag{7c}$$

and $\hat{P}$ satisfies

$$\hat{P}\hat{A}_0 + \hat{A}_0^T \hat{P} + 2I_n = 0 \tag{7d}$$

The conservatism of the condition with respect to the transformation can clearly be compared with Eq. (3) by using the index, defined by

$$\beta \triangleq (\mu - \epsilon)/\mu, \quad \hat{\beta} \triangleq (\hat{\mu} - \hat{\epsilon})/\hat{\mu} \tag{8a}$$

where it can be seen that $\beta > 0$ (or $\hat{\beta} > 0$) when the stability condition of Eq. (3) [or Eq. (7)] is satisfied and that

$$\hat{\beta} > \beta \tag{8b}$$

indicates the transformation to be effective in reducing the conservatism of the condition. Let us illustrate this by an example.

Example 1: Let A_0 of Eq. (1) be given by

$$A_0 = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}$$

Suppose that only element a_{11} gets perturbed and that

$$\epsilon_{11} = |e_{11}(t)|_{\max} = 2$$

Then clearly

$$\epsilon = 2 \quad \text{and} \quad U_e = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

The right-hand side of Eq. (3) gives $\mu = 1.657$. Thus

$$\beta = (1.657 - 2)/1.657 = -0.207$$

is negative indicating that the stability condition is not satisfied.

However, employing the transformation matrix

$$Q = \text{diag}[1, 1000]$$

the transformed quantities are

$$\hat{\epsilon} = 2, \quad \hat{U}_e = U_e, \quad \text{and} \quad \hat{\mu} = 3$$

Thus

$$\hat{\beta} = (3-2)/3 = 0.333$$

indicating that the stability condition is satisfied and hence the system (even in the original coordinates) is stable. Thus, the use of transformation reduced the conservatism of the stability condition.

Case 2: Left-Hand Side Is Not Known (Specifying the Perturbation Bound)

As before in case 1, after the transformation, the stability condition is

$$\hat{\epsilon} < \hat{\mu} \tag{9}$$

However, here the bound $\hat{\mu}$ is given on the transformed perturbation $\hat{\epsilon}$ and not on the original perturbation ϵ. Evidently, in order to examine the usefulness of the transformation in getting an improved bound, it is necessary to obtain the bound on ϵ *after* the transformation. Let μ^* denote the bound on ϵ after the transformation. Then, we have the following result.

Given $Q = \text{diag}[q_1, q_2, \ldots, q_n]$ and Eq. (6), the system of Eq. (1) is stable if

$$\epsilon < \mu^* \tag{10a}$$

$$\mu^* = \hat{\mu}\frac{1}{U_{e_{rs}}}\left|\frac{q_r}{q_s}\right| \tag{10b}$$

where $\hat{\mu}$ is given by Eq. (7) and r, s are such that rs is the specific entry in $\hat{U}_e$ corresponding to $U_{eij\max}$ where $\hat{U}_e = (Q^{-1})_m U_e Q_m$.

Clearly $\mu^* > \mu$ indicates the reduction in conservatism of the condition.

It can also be noted that

$$\mu^* > \mu \tag{11a}$$

if and only if

$$\hat{\beta} > \beta \tag{11b}$$

where $\hat{\beta}$ and β are given by Eq. (8).

Example 2: Let us again consider

$$A_0 = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}$$

and let

$$U_e = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Then we have

$$\mu = 0.3972$$

By using the transformation $Q = \text{diag}[1, 1.8]$ it can be shown that

$$\mu^* = 0.473$$

indicating around 20% improvement in the bound.

Some comments concerning the determination of the transformation matrix Q of Eq. (5) are now in order. It may be seen from the above examples that the transformation is indeed effective in considerably reducing the conservatism (either getting $\hat{\beta} > \beta$ or $\mu^* > \mu$). The specific transformation matrices obtained in these examples utilized the structural information about the nominal and perturbation matrices (A_0 and U_e, respectively). Clearly, there exists a specific transformation that maximizes the measure of reduction in conservatism (either $\hat{\beta}$ or μ^*) for any given problem at hand; also, this specific transformation very much depends on the structural information available for the particular example being considered. The ideal way to systematically determine the transformation would be to pose the problem as a parameter optimization problem [e.g., maximizing μ^* (or $\hat{\beta}$) with respect to q_i]; an analytical or algorithmic solution to this problem is not a trivial task. Even if a solution is found for the *general* problem, the benefits accrued by the method for a *specific* problem may be minimal. So before embarking on a general procedure to find the transformation, it is sometimes worthwhile to employ an ad hoc computer search for the parameters q_i to obtain improved results for a specific problem, as the following VTOL aircraft control example will illustrate. Even though the procedure for finding the transformation is ad hoc in these examples, the following guidelines are given to aid the search.

Briefly, for case 1, the computer search for the transformation Q is done as follows:

1) Take $Q = I_n$ (I_n is an $n \times n$ identity matrix).

2) Keep $q_l = 1$ and q_k ($k = 3, 4, \ldots, n$) $= 1$ and vary q_2 in a region of positive real numbers until $\hat{\beta}$ reaches a high value (hopefully the maximum value possible).

3) Fix q_2 at the value obtained in step 2 (with q_1 still $= 1$) and repeat the procedure in step 2 with q_3, keeping q_k ($k = 4, 5, \ldots, n$) $= 1$.

4) Repeat steps 1-3 for all other q_k ($k = 4, 5, \ldots,$).

An effort to determine the diagonal transformation Q more systematically is presented in Ref. 24, which incidentally treats not only structural (elemental) bound improvement (as is the case with this paper), but also unstructured (matrix norm) bound improvement as well.

It is to be pointed out at this stage that the main focus of the present paper in general (and the next section in particular) is to demonstrate the practical implications and utility of the transformation technique in designing robust controllers for a given application. Toward this direction, we exploit the role of transformation in obtaining a possible reduction in conservatism to present a simple constant *linear* state feedback control law for the VTOL aircraft control problem, widely considered in the literature,[6,25] that guarantees stability for the entire range of parameter variations reported for that particular problem. Recall that Ref. 6 suggests an adaptive control algorithm for the given range of perturbations, while Ref. 25 recommends a nonlinear feedback control that, however, cannot guarantee robustness for the *entire* range of perturbations reported.[6] In the next section, we present a simple constant linear state feedback control gain using the perturbation bound analysis with state transformation.

III. Application to VTOL Aircraft Control

The linearized model of the VTOL aircraft in the vertical plane is described by

$$\dot{x}(t) = [A_0 + \Delta A(t)]x(t) + [B_0 + \Delta B(t)]u(t) \tag{12}$$

The components of the state vector $x \rightarrow R^4$ and the control vector $u \rightarrow R^2$ are given by

$x_1 \rightarrow$ horizontal velocity, knots

$x_2 \rightarrow$ vertical velocity, knots

$x_3 \rightarrow$ pitch rate, deg/s

$x_4 \rightarrow$ pitch angle, deg

$u_1 \rightarrow$ "collective" pitch control

$u_2 \rightarrow$ "longitudinal cyclic" pitch control

In Ref. 6, it is shown that significant changes take place only in the elements a_{32}, a_{34}, and b_{21}. The ranges of values taken by these elements are given[6] as

$$\begin{aligned} 0.0663 &\le \bar{a}_{32}(\simeq 0.3681) \le 0.5044 \\ 0.122 &\le \bar{a}_{34}(\simeq 1.422) \le 2.528 \\ 0.977 &\le \bar{b}_{21}(\simeq 3.544) \le 5.1114 \end{aligned} \tag{13}$$

where $(\bar{\cdot})$ denotes the nominal value.

Note that the perturbation ranges are asymmetric with respect to the nominal values. In order to take full advantage of the perturbation bound analysis presented in Sec. II, we will "bias" the nominal value of a_{32}, a_{34}, and b_{21} such that we obtain symmetric bounds. Accordingly, the nominal values of a_{32}, a_{34}, and b_{21} now are $\bar{a}_{32} = 0.2855$, $\bar{a}_{34} = 1.3229$, and $\bar{b}_{21} = 3.04475$. The full matrices A_0 and B_0 are given by

$$A_0 = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.2855 & -0.707 & 1.3229 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_0^T = \begin{bmatrix} 0.4422 & 3.04475 & -5.52 & 0 \\ 0.1761 & -7.5922 & 4.49 & 0 \end{bmatrix}$$

so that

$$|\Delta A_{32}|_{\max} = 0.2197 \tag{14a}$$

$$|\Delta A_{34}|_{\max} = 1.2031 \tag{14b}$$

$$|\Delta A_{ij}| = 0 \text{ for all other } i \text{ and } j \tag{14c}$$

$$|\Delta B_{21}|_{\max} = 2.06725 \tag{14d}$$

$$|\Delta B_{ij}|_{\max} = 0 \text{ for all other } i \text{ and } j \tag{14e}$$

A robust constant-gain linear state feedback control law is obtained as follows:

Step 1: Taking ζ as a design variable, a Riccati-based feedback gain G_1 is obtained as

$$G_1 = \frac{1}{\zeta} B_0^T K \tag{15a}$$

where

$$KA_0 + A_0^T K - \frac{KB_0B_0^TK}{\zeta} + I_4 = 0 \tag{15b}$$

The closed-loop nominal matrix

$$\bar{A} = A_0 + B_0 G_1 \tag{16}$$

is asymptotically stable, as the conditions of complete controllability and observability are satisfied.

The perturbed closed-loop system is then given by

$$\dot{x}(t) = [(A_0 + B_0G_1) + (\Delta A + \Delta BG_1)]x(t) \tag{17a}$$

$$= [(A_0 + B_0G_1) + E]x(t) \tag{17b}$$

where $E_m = \Delta A_m + \Delta B_m G_{1m}$. Let $\epsilon_{ij} \triangleq E_{mij}$.

Step 2: Clearly, in this case, the perturbation matrix is fully known. The variable ζ is varied such that the stability robustness index

$$\beta \triangleq \frac{\mu(\zeta) - \epsilon(\zeta)}{\mu(\zeta)} \tag{18}$$

is made as large as possible where μ is given by

$$\mu = 1/\sigma_{\max}[P_m U_e]s \tag{19a}$$

and P_m satisfies

$$P(A_0 + B_0G_1) + (A_0 + B_0G_1)^T P + 2I_n = 0 \tag{19b}$$

[Note that both μ and ϵ are functions of gain G_1 (and ζ).]

The value of ζ that makes β of Eq. (18) maximum for the given perturbations of Eq. (14) is obtained as $\zeta = 3.6$. The corresponding gain G_1 is given by

$$G_1 = \begin{bmatrix} -0.467 & 0.01388 & 0.539 & 0.806 \\ 0.043 & 0.3828 & -0.1899 & -0.5947 \end{bmatrix} \tag{20}$$

and the corresponding value of β is given by

$$\beta = -0.21$$

Also, no diagonal transformation Q could be found such that $\hat{\beta} > 0$, which shows that the stability robustness condition is not satisfied with the gain G_1 of Eq. (20) and which, it may be recalled, is a Riccati-based gain.

Step 3: Evidently, one needs to look for a gain such that β (or $\hat{\beta}$) is positive. In other words, one needs to increase the bound μ without much increase in ϵ in Eq. (18). Toward this direction, for this particular example, we first propose to use a gain G_2 such that

$$\tilde{E}_m = \Delta A_m + \Delta B_m G_{1m} + \Delta B_m G_{2m} = E_m \quad (\text{i.e., } \Delta B_m G_{2m} = 0) \tag{21}$$

Hence, $\tilde{\epsilon}_{ij} = \epsilon_{ij}$.

One form for G_2 that satisfies Eq. (21) (since only ΔB_{21} is nonzero) is

$$G_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ g_{21} & g_{22} & g_{23} & g_{24} \end{bmatrix}$$

The motivation behind the selection of a specific G_2 matrix as above is to make the norm of the matrix

$$\tilde{A} = A_0 + B_0G_1 + B_0G_2 = A_0 + B_0G \quad (G \triangleq G_1 + G_2) \quad (22)$$

bigger with the hope of decreasing the norm of the Lyapunov matrix $\tilde{P}$, where $\tilde{P}$ satisfies

$$\tilde{P}\tilde{A} + \tilde{A}^T\tilde{P} + 2I_n = 0 \quad (23)$$

which, in turn, may help increase the bound μ. With this in mind, we take $g_{21} = g_{23} = 0$ and $g_{22} > 0$ and $g_{24} < 0$. For simplicity, we choose $|g_{22}| = |g_{24}|$.

To guarantee the asymptotic stability of $A_0 + B_0G$ of Eq. (22), we could think of the matrix B_0G_2 as a perturbation on the nominal stable matrix $A + B_0G_1$ and apply the perturbation bound condition of Eq. (3). With this done, we get

$$7.6g_{22} < 1.0342$$

which makes $g_{22} = 0.1362$ and $g_{24} = -0.1362$.

Thus, we finally get

$$G = G_1 + G_2 = \begin{bmatrix} -0.467 & 0.01388 & 0.539 & 0.806 \\ 0.043 & 0.519 & -0.1899 & -0.731 \end{bmatrix} \quad (24)$$

and

$$A_0 + B_0G = \begin{bmatrix} -0.2356 & 0.1246 & 0.22377 & -0.2277 \\ -1.7021 & -4.908 & 3.0859 & 3.98 \\ 2.8732 & 2.539 & -4.5359 & -6.408 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Now the computation of β with this new closed-loop system gives

$$\beta_{\text{new}} = -0.1722 \quad (25)$$

which is still negative but is an improvement over the β obtained before. However, one cannot conclude the stability of the perturbed closed-loop system based on this β_{new}.

Step 4: We now apply a state transformation for the above nominal system using a transformation matrix

$$Q = \text{diag}[1.0, 2.3, 1.46, 1.14] \quad (26)$$

The computation of $\hat{\beta}_{\text{new}}$ (after the transformation) gives

$$\hat{\beta}_{\text{new}} = 0.029 > 0$$

Thus, the state transformation given by the matrix Q of Eq. (26) has reduced the conservatism of the stability condition and the gain matrix given by Eq. (24) guarantees robust asymptotic stability of the system in the entire range of parameter perturbations considered in Ref. 6.

IV. Conclusions

In this paper, the role of state transformation to obtain less conservative perturbation bounds in the stability robustness analysis of linear systems with structured (elemental) uncertainty is illustrated. The transformation is such that it uses the structural information about the nominal as well as perturbation matrices. By combining the state transformation technique with the recently reported method of obtaining elemental bounds, it was possible to design a simple constant linear state feedback control gain for a VTOL aircraft control problem such that the gain stabilizes the system over the entire range of elemental perturbations considered.

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